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Heat Transfer Analysis on a Moving Flat Sheet Emerging into Quiescent Fluid

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The conjugated heat conduction process of a continuously moving flat sheet emerging from a slot or orifice in contact with a quiescent fluid is analyzed. Because of the finite thermal conductivity of the sheet, longitudinal and transverse temperature gradients arise within it and, thus, change the mathematical character of the problem from parabolic to elliptic. The momentum and energy balance equations are reduced to a nonlinear system of partial differential equations with three parameters: the Prandtl number Pr, a nondimensional sheet thermal conductance β, and a suitable Peclet number Pe. The limits $\beta \ll 1$ and $\beta Pe^2 \ll 1$ are identified as the most relevant from a practical point of view. In this case, the problem is governed by an universal integral equation to obtain the spatial evolution of the sheet temperature as a function of the nondimensional longitudinal coordinate.

Nomenclature

c_s = specific heat of the sheet material
f = nondimensional stream function introduced in Eq. (13)
h = half-thickness of the sheet
L = cooling distance of the sheet
L^* = thermal penetration length, α_s/U
Nu = Nusselt number defined in Eq. (9)
Pe = Peclet number defined by hU/α_s
Pr = Prandtl number of the cooling fluid
Re = Reynolds number, $\rho_f U/\nu_f$
T_f = temperature of the cooling fluid
T_s = temperature of the sheet
T_0 = initial temperature of the sheet
T_1 = temperature of the sheet at the exit
T_∞ = temperature of the cooling fluid far from the sheet
U = velocity of the sheet
u, v = nondimensional longitudinal and transversal velocities defined in Eq. (13)
x, y = Cartesian coordinates
z = nondimensional transversal coordinate of the plate defined by y/h
α_f = thermal diffusivity of the cooling fluid
α_s = thermal diffusivity of the sheet material
β = longitudinal heat conduction parameter defined in Eq. (5)
δ = thickness of the thermal boundary layer
ζ = nondimensional inner coordinate defined by $\beta\chi$
η = nondimensional transversal coordinate for the cooling fluid defined in Eq. (13)
θ = nondimensional temperature of the cooling fluid
θ_s = nondimensional temperature in the sheet
λ_f = thermal conductivity of the cooling fluid
λ_s = thermal conductivity of the sheet material
ν_f = kinematic coefficient of viscosity of the cooling fluid
ρ_s = density of the sheet material
χ = nondimensional longitudinal coordinate defined by x/L^*

Subscripts

f = cooling fluid
s = sheet

I. Introduction

THE conjugated heat transfer process with a boundary-layer behavior on a moving continuous solid surface through a specific medium is an important mechanism that occurs frequently in many branches of engineering applications. In fact, technological examples of such physical situations can be found in petroleum drilling, glass and paper production, extrusion of a polymeric or complex metal sheet, drawing of thin plastic films, among others. In all of these studies, the heat transfer rates play a fundamental role to control the final quality of the final products. We give special attention to isolating the conjugate heat transfer analysis due to the large number of extended physical parameters. The conjugate problem is described by the thermal interaction between the moving sheet and the adjacent thermal boundary layer of the fluid. When this condition is introduced, it is then necessary to consider nonisothermal conditions in the sheet to have an adequate description of the involved phenomena. In well-recognized works, the specialized literature for moving sheets with thermal boundary-layer processes has paid considerable attention for the classical schemes for isothermal conditions reported in the past. Since the pioneering papers of Sakiadis[1] and Chida and Kato,[2] simplifications using similarity solutions have been reexamined during the past decades to improve understanding about this process. We mention Ali[3] and Grubka and Bobba,[4] who, among others, have representative studies of these schemes. In these analyses, the thermal conditions frequently are modeled by different power-law variations of the temperature or flux distribution for the sheet. (The isothermal moving continuous surface is understood here as a particular case of power laws.) A recent work by Magyari and Keller[5] summarizes the state of the art of several physical configurations and identifies similarity solutions to describe the steady boundary layers on an exponentially stretching continuous surface with an exponentially temperature distribution. Similar studies, using moving plates embedded in porous medium, have been recently developed. In the past, the works of Elbashbeshy and Bazid[6] and Vafai and Tien[7] have received special mention.

Although the foregoing works are essential contributions, they are only reserved for cases with prescribed thermal boundary conditions. However, we emphasize that this situation is only valid for very idealized cases. This was recognized by Char et al.[8] in their numerical study of the laminar conjugate forced convection from a continuous moving sheet. Here, the thermal boundary-layer flow is coupled with the longitudinal heat conduction within the sheet. They mainly concluded that, for a given fluid, a decrease in the

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thermal conductivity of the moving plate results in a decrease in the temperature in the axial direction of the sheet.

In an effort to extend new solutions, where nonisothermal conditions are required, in this work we analyze practically the same conjugate laminar configuration reported by Char et al.[8] However, in the present model we consider two basic modifications: The temperature of the orifice or slot is not known a priori, and the characteristic lengths of the physical model are properly defined to avoid erroneous interpretations with the systematic use of dimensionalized variables. The heat flux from the sheet to the fluid is influenced strongly by the presence of the extended moving surface with finite thermal conductivity, when the longitudinal and transverse heat transfer conduction effects are taken into account. This thermal coupling between the horizontal moving sheet and the fluid phase offers a new theoretical extension to the fundamental previous works in laminar heat transfer with moving surfaces. In this work, using the boundary-layer approach for the stagnant fluid flow, we show that the longitudinal and transversal heat conduction through the plate depends mainly on three nondimensional parameters: the Prandtl number Pr, the parameter β, which takes into account the influence of the longitudinal heat conduction along the sheet, and the Peclet number Pe associated with the thermal diffusivity of the sheet material. In the majority of practical cases, the values of β are very small compared with unity. The influence of the Peclet number has serious consequences over the transverse and longitudinal heat transfer rates of the sheet. We anticipate that the ratio $\beta Pe^2 \ll 1$ is the relevant limit with $\beta \ll 1$, and we identify it as the thermally thin-sheet limit.

II. Formulation and Order of Magnitude Analysis

The physical model under study is shown in Fig. 1. A thin, horizontal plate or sheet of thickness $2h$ with uniform velocity U is continuously emerging from a slot and moving from left to right toward the semi-infinite stagnant environment of a cooling fluid of temperature T_∞. The thermal influence of the fluid is to modify drastically the longitudinal and transverse heat transfer rates along the sheet. Far away from the slot to the left, the temperature of the sheet is uniform, $T_s = T_0$, which is assumed to be larger than T_∞. The origin of the adopted coordinate axes is stationary and is located just at the slot. The y axis points out in the direction normal to the upper surface of the sheet ($y = 0$), and the longitudinal x axis points out in the direction of the motion of the thin sheet. In the region $x \le 0$, the moving sheet is surrounding by an adiabatic rigid enclosure. The heat flux along the sheet exists even before the plate emerges from the slot. The thermal penetration length to the left must be of order $L^* \sim \alpha_s/U$, where α_s is the thermal diffusivity of the sheet. As the sheet enters into contact with the cooling fluid at $x = 0$, because of the assumed finite thermal conductivity of the sheet material, the heat conduction in both the transverse and longitudinal directions is important because of the strong thermal influence of the convective environment. The sheet reaches the ambient temperature at values of $x \sim L$, where L is a characteristic length in the horizontal direction to be obtained through the analysis. Because of the symmetry of the physical model, we consider only the upper side of the moving sheet.

When a global energy balance is applied for $x \ge 0$, the thermal energy transferred from the moving sheet is convected to the cooling fluid. Using an order of magnitude estimate, this relationship can be written as

$$\rho_s c_s h U(T_1 - T_\infty) \sim \lambda_f L(T_1 - T_\infty)/\delta \tag{1}$$

where T_1 is the characteristic temperature at the slot position, to be determined later, and δ is the thickness of the thermal layer in the cooling fluid. For large values of Reynolds number, $Re = UL/\nu_f$, the thickness of the thermal boundary layer is given as

$$\delta \sim L/(Pr\,Re)^{\frac{1}{2}} \tag{2}$$

where $Pr = \nu_f/\alpha_f$ and $\alpha_f = \lambda_f/\rho_f c_f$. Here, ρ_f and c_f are the density and the specific heat of the fluid, respectively. Combining relationships (1) and (2), we obtain the order of magnitude for the characteristic length L as

$$L \sim h\,Pe(\alpha_f/\alpha_s)(\lambda_s/\lambda_f)^2 \tag{3}$$

where $Pe = Uh/\alpha_s = h/L^*$. As a consequence of this scale analysis, it is also significant to estimate the order of magnitude of the characteristic temperature drop in the transverse direction of the sheet ΔT_{sh}. The order of magnitude of the heat flux through the thin sheet up to the cooling fluid can be written as

$$\lambda_s(\Delta T_{\text{sh}}/h) \sim \lambda_f(\Delta T_f/\delta) \sim \lambda_f(\Delta T_f/L)(Re\,Pr)^{\frac{1}{2}} \tag{4}$$

where ΔT_f is the characteristic temperature difference in the cooling fluid. Then, $\Delta T_f + \Delta T_{\text{sh}} \sim T_1 - T_\infty$. Therefore,

$$\Delta T_{\text{sh}} \sim \frac{T_1 - T_\infty}{1 + \beta(L/h)^2}, \qquad \beta = \frac{1}{\pi}\frac{\lambda_s}{\lambda_f}\frac{h}{L}\frac{1}{(Pr\,Re)^{\frac{1}{2}}} = \frac{1}{\pi}\frac{L^*}{L} \tag{5}$$

where the nondimensional parameter β measures the effect of the longitudinal heat conduction in the process. For values of $\beta L^2/h^2 \sim 1/(\beta Pe^2) \gg 1$, the temperature variations in the transverse direction of the sheet are very small compared with the overall temperature difference $T_1 - T_\infty$, $\Delta T_{\text{sh}} \ll T_1 - T_\infty$. This regime corresponds to the thermally thin regime. Using the continuity of the temperature and its gradient at $x = 0$, we obtain

$$(T_0 - T_1)/L^* \sim (T_1 - T_\infty)/L \tag{6}$$

then the temperature of the sheet at $x = 0$ is given approximately by

$$T_1 \sim \frac{T_0 + \beta T_\infty}{1 + \beta} \tag{7}$$

For very small values of β compared with unity, the temperature of the sheet at the slot is very close to the initial temperature of the sheet, and the cooling rate is dictated mainly from the interaction with the cooling fluid. On the other hand, for very large values of β compared with unity, the temperature at the slot is close to the temperature of the cooling fluid, and the heat transfer process takes place mainly along the sheet to the left of the slot position.

The heat transfer rate due to the cooling process can then be written as

$$q = -\lambda_w \frac{\partial T}{\partial x}\bigg|_{x=0} \sim \frac{\lambda_w(T_0 - T_\infty)}{L^*}\frac{\beta}{1 + \beta} \tag{8}$$

or in nondimensional form

$$Nu = \frac{qL^*}{\lambda_w(T_0 - T_\infty)} \sim \frac{\beta}{1 + \beta} \tag{9}$$

For small values of β compared with unity, but still much larger then $(h/L)^2$, that is, $\beta Pe^2 \ll 1$, the temperature in the sheet is mainly a function of the longitudinal coordinate (thermally thin regime), and the temperature at the slot position is close to T_0, as already mentioned. The longitudinal heat conduction along the sheet is weak, and the heat transfer process is driven by convection by the moving sheet. We anticipate that, for practical applications, the values of β are always very low, of the order $\beta \sim 10^{-6}$, together with values of Peclet number Pe of the order unity. Therefore, the parameter βPe^2 is still very small compared with unity, and it has a large influence in modulating the conjugate heat transfer process. From relationships

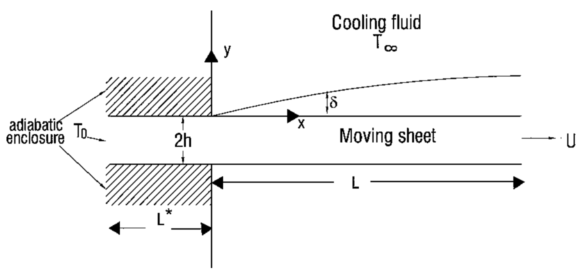

Fig. 1 Schematic diagram of the physical model investigated.

(3) and (5), we obtain that βPe^2 is independent of the sheet velocity and depends mainly on the transport coefficients of the sheet material and the cooling fluid as

$$\beta Pe^2 \sim \alpha_s/\alpha_f(\lambda_f/\lambda_s)^2 \tag{10}$$

The order of magnitude for the Reynolds number is then given by

$$Re = UL/\nu_f \sim (1/\beta)\alpha_s/\nu_f \tag{11}$$

which is very large compared with unity for very small values of β, thus justifying the boundary-layer approximation.

III. Governing Equations

When the following nondimensional variables for the sheet

$$\theta_s = (T_s - T_\infty)/(T_1 - T_\infty), \qquad \chi = x/L^*, \qquad z = y/h \tag{12}$$

and for the convective flow

$$\eta = \frac{y\sqrt{Re^*}}{\sqrt{L^* x}}, \qquad u = U\frac{\mathrm{d}f}{\mathrm{d}\eta}, \qquad v = \frac{U\sqrt{L^*}}{\sqrt{Re^* x}}\frac{1}{2}\left(\eta\frac{\mathrm{d}f}{\mathrm{d}\eta} - f\right)$$

$$\theta = \frac{T - T_\infty}{T_1 - T_\infty} \tag{13}$$

are introduced, the energy equation for the sheet can be written as

$$\frac{\mathrm{d}^2\theta_s}{\mathrm{d}\chi^2} = \frac{\mathrm{d}\theta_s}{\mathrm{d}\chi} \qquad \text{for} \qquad \chi \le 0 \tag{14}$$

$$\frac{\partial^2\theta_s}{\partial\chi^2} + \frac{1}{Pe^2}\frac{\partial^2\theta_s}{\partial z^2} = \frac{\partial\theta_s}{\partial\chi} \qquad \text{for} \qquad \chi \ge 0 \tag{15}$$

The boundary conditions are

$$\theta_s(\chi \to -\infty) = \theta_0, \qquad \theta_s(\chi \to +\infty, z) \to 0 \tag{16}$$

together with

$$\theta_s|_{\chi=0^+} - 1 = \theta_s|_{\chi=0^-} - 1 = \left.\frac{\mathrm{d}\theta_s}{\mathrm{d}\chi}\right|_{\chi=0^-}$$

$$-\left.\frac{\partial\theta_s}{\partial\chi}\right|_{\chi=0^+} = \left.\frac{\partial\theta_s}{\partial z}\right|_{z=-1} = 0 \tag{17}$$

and suitable boundary conditions to satisfy the continuities of the temperature and heat flux between the external surface of the sheet and the fluid, to be given subsequently. In the preceding equations $\chi = 0^{+(-)}$ is the longitudinal coordinate evaluated from the right- or left-hand side of the slot. Re^* is the Reynolds number defined with the charachteristic penetration length as $Re^* = UL^*/\nu_f$. The corresponding momentum and energy equations for the fluid, using the boundary-layer approximation, are given by

$$\frac{\mathrm{d}^3 f}{\mathrm{d}\eta^3} + \frac{1}{2}f\frac{\mathrm{d}^2 f}{\mathrm{d}\eta^2} = 0 \tag{18}$$

$$\frac{1}{Pr}\frac{\partial^2\theta}{\partial\eta^2} + \frac{1}{2}f\frac{\partial^2\theta}{\partial\eta^2} = \chi\frac{\mathrm{d}f}{\mathrm{d}\eta}\frac{\partial\theta}{\partial\chi} \tag{19}$$

with the boundary conditions

$$\theta(\chi,0) - \theta_s(\chi,0) = f(0) = \left.\frac{\mathrm{d}f}{\mathrm{d}\eta}\right|_{\eta=0} - 1 = 0$$

$$\text{at} \quad z = \eta = 0 \tag{20}$$

$$\left.\frac{\partial\theta_s}{\partial z}\right|_{z=0} = Pe\frac{\lambda_f}{\lambda_s}\left(\frac{\alpha_s}{\nu_f}\right)^{\frac{1}{2}}\frac{1}{\sqrt{\chi}}\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0} \tag{21}$$

$$\frac{\mathrm{d}f}{\mathrm{d}\eta} = \theta = 0 \qquad \text{at} \qquad \eta \to \infty \tag{22}$$

Note that Eq. (18) subject to the boundary conditions (20) and (22) also describes the free convection boundary layer over an isothermal vertical flat plate embedded in a fluid-saturated porous medium, studied by Cheng and Minkowycz.[9]

The solution of problem (15–22) should provide

$$\theta_s = \theta_s(\beta, Pe, Pr, \chi, z)$$

System (14–22) can be easily simplified by solving Eq. (14) together with Eqs. (16) and (17), giving

$$\theta_s = \theta_0 + (1 - \theta_0)\exp(\chi) \qquad \text{for} \qquad \chi \le 0 \tag{23}$$

Using this solution, together with relationship (17) for the continuity condition of the heat flux at $\chi = 0$, we obtain a simple expression to evaluate T_1:

$$T_1 = \frac{T_0 - T_\infty(\partial\theta_s/\partial\chi)|_{\chi=0}}{1 - (\partial\theta_s/\partial\chi)|_{\chi=0}} \tag{24}$$

Thus, from the solution of the system of equations (15) and (17), we can compute, among other relevant quantities, the longitudinal heat flux at $\chi = 0^+$, which in nondimensional form is given by

$$Nu = \frac{qL^*}{\lambda_w(T_0 - T_\infty)} = \frac{-\partial\theta_s/\partial\chi|_{\chi=0}}{1 - \partial\theta_s/\partial\chi|_{\chi=0}} \tag{25}$$

Hereinafter we obtain the numerical and asymptotic solutions according to the assumed values of β.

IV. Thermally Thin Wall Regime ($\beta Pe^2 \ll 1$)

In the thermally thin-wall regime, the temperature in the sheet depends primarily on the longitudinal coordinate $\theta_s(\chi)$. The system of equations (15–17), together with Eq. (24) was nondimensionalized using the characteristic length L^*. This characteristic length appears directly in a first approximation as a natural choice. However, the foregoing system is not properly normalized to take into account the convective heat transfer to the cooling fluid. For this part of the system, $\chi \ge 0$, it is preferable to scale the longitudinal coordinate with L instead of L^*. We then introduce a new nondimensional coordinate $\zeta = \beta\chi \sim \mathcal{O}(1)$. Inasmuch as the energy equation for the sheet (15) can be integrated through the transverse coordinate, after applying the boundary conditions (17) and (21), we obtain

$$\beta\frac{\mathrm{d}^2\theta_s}{\mathrm{d}\zeta^2} = \frac{\mathrm{d}\theta_s}{\mathrm{d}\zeta} - \sqrt{\frac{\pi}{Pr\zeta}}\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0} \tag{26}$$

together with the nondimensional boundary conditions

$$\theta_s(0) = 1, \qquad \theta_s(\zeta \to \infty) \to 0$$

The energy equation for the cooling fluid, Eq. (19), transforms to

$$\frac{1}{Pr}\frac{\partial^2\theta}{\partial\eta^2} + \frac{1}{2}f\frac{\partial\theta}{\partial\eta} = \zeta\frac{\mathrm{d}f}{\mathrm{d}\eta}\frac{\partial\theta}{\partial\zeta} \tag{27}$$

whereas the nondimensional boundary conditions associated with the fluid governing equations are given by

$$\theta(\zeta,\eta) - \theta_s(\zeta) = f = \frac{\mathrm{d}f}{\mathrm{d}\eta} - 1 = 0 \qquad \text{at} \qquad \eta = 0 \tag{28}$$

$$\frac{\mathrm{d}f}{\mathrm{d}\eta} = \theta(\zeta,\eta) = 0 \qquad \text{for} \qquad \eta \to \infty \tag{29}$$

The system of equations (27–29) can be solved through the use of Lighthill's integral technique,[10] where the nondimensional temperature gradient at the surface of the sheet can be written as a function of the nondimensional temperature profile upstream of that position as

$$\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0} = -\sqrt{\frac{Pr}{\pi}}\left[\int_0^\zeta \frac{1}{(1 - t/\zeta)^{\frac{1}{2}}}\frac{\mathrm{d}\theta_s}{\mathrm{d}t}\mathrm{d}t + 1\right] \tag{30}$$

Therefore, Eq. (26) now takes the form

$$\beta\frac{d^2\theta_s}{d\zeta^2} = \frac{d\theta_s}{d\zeta} + \frac{1}{\sqrt{\zeta}}\left[1 + \int_0^{\zeta}\left(1-\frac{t}{\zeta}\right)^{-\frac{1}{2}}\frac{d\theta_s}{dt}dt\right] \tag{31}$$

which has to be solved with the boundary conditions $\theta_s(0) = 1$ and $\theta_s(\infty) \to 0$. The problem now reduces to solving a single integro–differential equation (31) for θ_s with only one parameter β. For very small values of β, which is very important from a practical point of view, the problem can be solved using the boundary-layer approximation. Close to the slot, in a region of order $\zeta \sim \beta$, the inner equation, where the longitudinal heat conduction along the sheet must be retained, takes the form

$$\frac{d^2\varphi}{d\sigma^2} = \frac{d\varphi}{d\sigma} - \frac{1}{\sqrt{\sigma}}\left[1 + \beta^{\frac{1}{2}}\int_0^{\sigma}\left(1-\frac{t}{\sigma}\right)^{-\frac{1}{2}}\frac{d\varphi}{dt}dt\right] \tag{32}$$

where

$$\varphi(\sigma) = (1-\theta_s)\big/\beta^{\frac{1}{2}} \qquad \text{with} \qquad \sigma = \zeta/\beta \tag{33}$$

The solution, to leading order, is given by

$$\varphi(\sigma) = 2\sigma^{\frac{1}{2}} - \sqrt{\pi} + \sqrt{\pi}\,\mathrm{erfc}\left(\sigma^{\frac{1}{2}}\right)e^{\sigma} \tag{34}$$

with the asymptotic behaviors

$$\varphi(\sigma) \sim \sqrt{\pi}\sigma - \tfrac{4}{3}\sigma^{\frac{3}{2}} + \tfrac{1}{2}\sqrt{\pi}\sigma^2 - \tfrac{8}{15}\sigma^{\frac{5}{2}} + \mathcal{O}(\sigma^3) \qquad \text{for} \quad \sigma \to 0 \tag{35}$$

$$\varphi(\sigma) \sim 2\sigma^{\frac{1}{2}} - \sqrt{\pi} + \sigma^{-\frac{1}{2}} \qquad \text{for} \qquad \sigma \to \infty \tag{36}$$

The nondimensional temperature gradient at $\zeta = 0$ is then given by

$$\left.\frac{d\theta_s}{d\zeta}\right|_{\zeta=0} = -\sqrt{\frac{\pi}{\beta}} \qquad \text{for} \qquad \beta \to 0 \tag{37}$$

Outside of the inner layer, the solution to the outer equation, given by Eq. (31) with $\beta = 0$, can be written for small values of ζ as

$$\theta_s^{(o)} = 1 - 2\zeta^{\frac{1}{2}} + \pi\zeta - \tfrac{4}{3}\pi\zeta^{\frac{3}{2}} + \tfrac{1}{2}\pi^2\zeta^2 + \mathcal{O}\left(\zeta^{\frac{5}{2}}\right) \tag{38}$$

Figure 2 shows the leading solution of the outer region represented by Eq. (31) with $\beta = 0$. For values of $\zeta > 4$, the temperature of the sheet practically reaches the temperature of the cooling fluid, T_∞. On the other hand, for large values of β, it can easily be shown that

$$\left.\frac{d\theta_s}{d\zeta}\right|_{\zeta=0} = -\frac{1.2034}{\beta^{\frac{2}{3}}} \qquad \text{for} \qquad \beta \to \infty \tag{39}$$

Figure 3 shows the nondimensional temperature gradient as a function of β, obtained from the numerical calculation of Eq. (31). It is clear from Fig. 3 that the leading-order asymptotic solution is valid for values of $\beta \lesssim 10^{-3}$.

The numerically computed nondimensional heat flux at the slot is shown in Fig. 4, using the thermally thin-wall regime. We also present in Fig. 4 the asymptotic limits for very small and very large values of β compared with unity, which are given by

$$Nu = \frac{-\beta\, d\theta_s/d\zeta|_{\zeta=0}}{1 - \beta\, d\theta_s/d\zeta|_{\zeta=0}} \simeq \begin{Bmatrix} \dfrac{\sqrt{\pi\beta}}{1+\sqrt{\pi\beta}} & \beta \to 0 \\ \dfrac{1.2034\beta^{\frac{1}{3}}}{1+1.2034\beta^{\frac{1}{3}}} & \beta \gg 1 \end{Bmatrix} \tag{40}$$

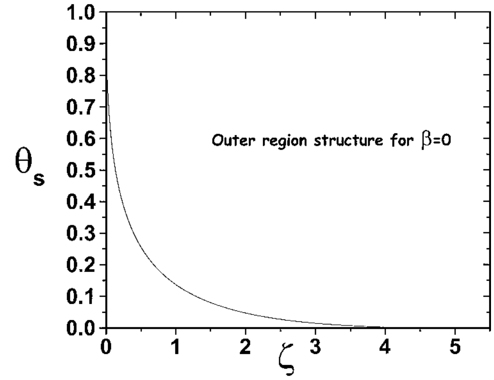

Fig. 2 Thermal structure of the outer region, obtained numerically from Eq. (31) with $\beta = 0$.

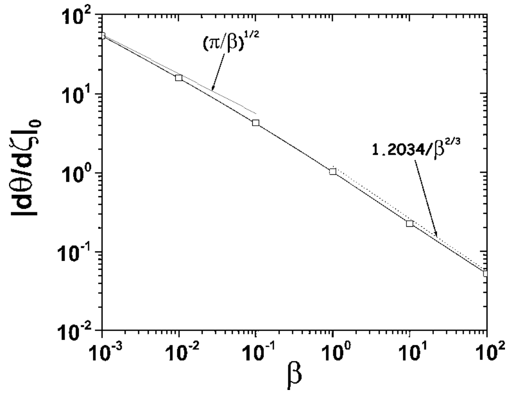

Fig. 3 Nondimensional temperature gradient in the moving sheet at $\zeta = 0$, for different values of β, and asymptotic behaviors for small and large values of β compared with unity.

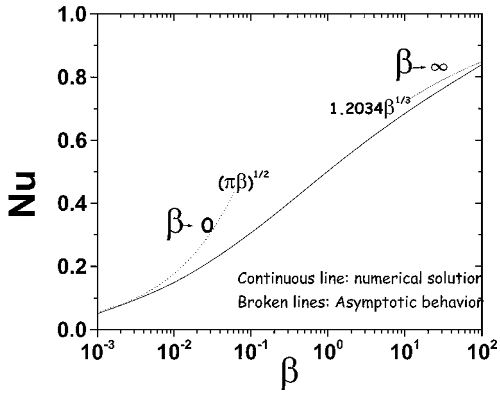

Fig. 4 Nusselt number for different values of β and asymptotic behaviors for small and large values of β compared with unity.

V. Results

Equation (31) has been numerically integrated using a conventional fourth-order Runge–Kutta technique by transforming the boundary value to an initial value problem.[11,12] The unknown temperature at the slot, T_1, can be easily obtained from Eq. (24). In Fig. 5, we show the normalized nondimensional temperature $(T_s - T_\infty)/(T_0 - T_\infty)$ for three values of the nondimensional parameter $\beta = 0.1$, 1, and 10, as a function of the nondimensional longitudinal coordinate χ scaled with L^*. As predicted by the order of magnitude analysis, for decreasing values of the parameter β, the nondimensional temperature tends to reach an uniform asymptotic value of $(T_s - T_\infty)/(T_0 - T_\infty) \sim 1$, which is equivalent to imposing an uniform sheet temperature. In this case, the conjugate heat transfer problem is practically attenuated, and we recover the solution of Sakiadis.[1] Otherwise, for increasing values of β, the nondimensional

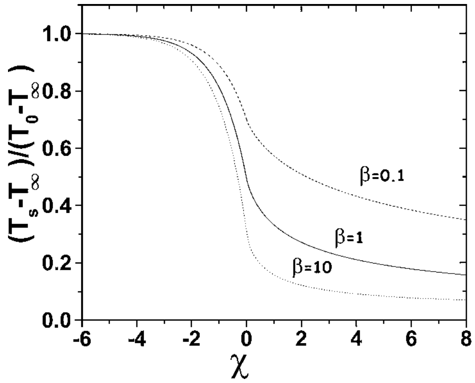

Fig. 5 Normalized temperature in the moving sheet as a function of the nondimensional longitudinal coordinate $\chi = x/L^*$, for three different values of β.

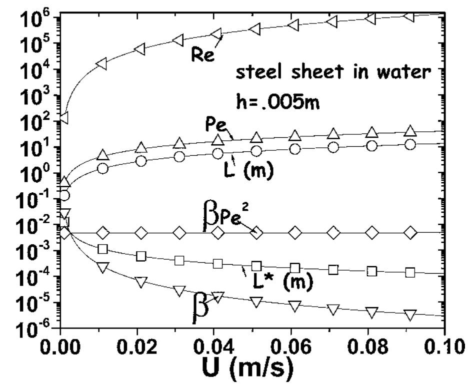

Fig. 6 Nondimensional parameters *Re*, *Pe*, β, and βPe^2 and characteristic thermal lengths, *L* and L^*, for a steel sheet cooled in water as a function of the sheet velocity with $h = 0.005$ m.

temperature $(T_s - T_\infty)/(T_0 - T_\infty)$ appears to be similar to the Heaviside unit step function, that is, for $x \leq 0$, the temperature is not substantially modified by the value T_0. However, as $x > 0$, the temperature $(T_s - T_\infty)/(T_0 - T_\infty)$ is drastically modified by this value, and particularly, T_s reaches the asymptotic value of $T_s \sim T_\infty$. In this latter case, the sheet temperature easily is found in thermodynamic equilibrium with the stagnant medium. Here, a strong and sudden heat transfer process between the sheet and the fluid is carried out for distances very close to the orifice. Note that for the $\beta = 1$ case, $(T_s - T_\infty)/(T_0 - T_\infty)$ is close to 0.5 at $\zeta = 0$, the point where the heat transfer rate is abruptly inverted due to the cooling process with the stagnant fluid.

To clarify the roles of the nondimensional parameters and to justify the assumptions made in the analysis, we did a numerical example of a steel moving sheet emerging in a water tank, using two different sheet thickness. Figures 6 and 7 show the resulting values of the important nondimensional parameters *Re*, *Pe*, β, and βPe^2 as a function of the sheet velocity, up to 0.1 m/s. The characteristic thermal lengths L and L^* are also plotted in Figs. 6 and 7. The Reynolds numbers are always very large compared with unity, which justifies the use of the boundary-layer approximation, except for very small values of the sheet velocity, as shown in Figs. 6 and 7. As we mentioned earlier, the Peclet number is of the order unity in the cases analyzed. The corresponding values of β are also very small compared with unity, showing that the longitudinal heat conduction has to be retained in a thin zone close to the slot position. The asymptotic formulas for the temperature and heat flux obtained in the limit of $\beta \to 0$ applied in these cases. The values of βPe^2 are always very small compared with unity and do not depend on the sheet velocity, as can be shown easily. Therefore, the thermally thin-sheet regime clearly applies to these cases. In addition, a relevant parameter in this type of cooling processes is represented by the characteristic

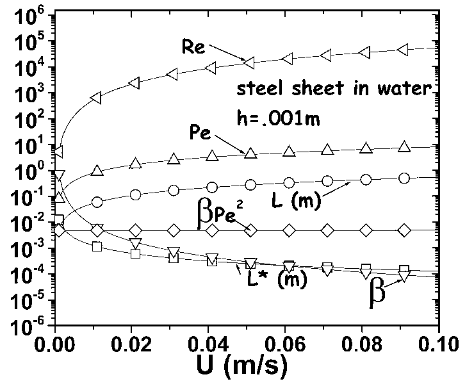

Fig. 7 Nondimensional parameters *Re*, *Pe*, β, and βPe^2 and characteristic thermal lengths, *L* and L^*, for a steel sheet cooled in water as a function of the sheet velocity with $h = 0.001$ m.

cooling time, which has a strong influence on a potential thermal shock. In our physical model, this characteristic time is of order

$$t_c \sim \alpha_s \big/ U^2[(1+\beta)/\beta] \tag{41}$$

which for very small values of β compared with unity is given by

$$t_c \sim \left(h^2 \big/ \alpha_s\right) \alpha_f / \alpha_s (\lambda_s/\lambda_f)^2 \tag{42}$$

which is also independent of the sheet velocity.

VI. Conclusions

In this paper we have studied both analytically and numerically the conjugated heat transfer of a convective cooling of a moving sheet. The sheet, initially at a temperature T_0, is moving with a uniform velocity toward a bath of cooling fluid at temperature T_∞. The temperature evolution of the sheet has been obtained, and the main nondimensional parameters have been identified. We have shown that to analyze properly the laminar conjugate forced convection from a continuous moving sheet, it is necessary to identify the roles of the two characteristic thermal lengths, L^* and L, directly related to the process. In fact, the parametric influence of these lengths is represented through the use of the nondimensional parameter β, which measures the effect of the longitudinal heat conduction along the sheet. On the other hand, the parameter βPe^2 controls the appropriate thermal regime. In practical situations, the values of β and Peclet number *Pe* are $\beta \sim 10^{-6}$ and $Pe \sim 1$, and, therefore, the limit $\beta Pe^2 \ll 1$ appears to be the most relevant case and corresponds to the thermally thin-sheet regime. Thus, in a first approximation, the temperature variations in the transverse direction of the sheet are very small compared with the overall temperature difference $T_1 - T_\infty$ and its dependence with the transverse coordinate can be neglected. In this regime, the conjugate heat transfer problem is reduced to solving a single integro-differential equation (31) for the temperature of the moving sheet, with a single free parameter corresponding to β. The limiting regime of a thermally thin sheet has been studied in this work, assuming a large value of the Reynolds number compared with unity. For large values of β compared with unity, the heat transfer process takes place mainly inside the adiabatic enclosure, and the corresponding temperature of the sheet material at the exit of the slot is close to the fluid temperature. On the other hand, for very small values of β compared with unity, which is of practical interest, the heat transfer process takes place after reaching the exit position, where the temperature of the sheet is close to the initial temperature T_0.

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